

Ա. Ի. ԱԼԻԿԽԱՆՅԱՆԻ ԱՆՎԱՆ ԱԶԳԱՅԻՆ ԳԻՏԱԿԱՆ ԼԱԲՈՐԱՏՈՐԻԱ  
(ԵՐԵՎԱՆԻ ՖԻԶԻԿԱՅԻ ԻՆՍՏԻՏՈՒՏ)

Ապրեսյան Ելենա Անդրանիկի

ՎՈՒՄԻ ԲՎԱՆՏԱՅԻՆ ԵՐԵՎՈՒՅՅԻ ՈՐՈՇ ԱՍՊԵԿՏՆԵՐԸ

01 04 02 «Տեսական ֆիզիկա» մասնագիտությամբ ֆիզիկամաթեմատիկական  
գիտությունների բնագավառի գիտական առարկաների հայրենական  
առնչությունները

ՍԵՂՍԱԳԻՐ

ԵՐԵՎԱՆ – 2019

---

A I ALIKHANYAN NATIONAL SCIENCE LABORATORY  
(YEREVAN PHYSICS INSTITUTE)

Apresyan Yelena

NUMERICAL ASPECTS OF QUANTUM HALL EFFECT

SYNOPSIS

of Dissertation in 01 04 02 Theoretical Physics Presented for the degree of  
candidate in physical and mathematical Sciences

YEREVAN 2019

Ատենախոսության թեման հաստատվել է Ա. Ի. Ալիխանյանի անվան Ազգային Գիտական Լաբորատորիայի (ԵրՖի) գիտական խորհուրդում:  
Գիտական ղեկավար՝

Ֆիզ.-մաթ գիտ.թեկնածու Ա.Սեդրակյան(ԱԱԳԼ)

Պաշտոնական ընդդիմախոսներ՝

Ֆիզ. մաթ գիտ. դոկտոր Ռ.Պողոսյան(ԱԱԳԼ)

Ֆիզ.-մաթ գիտ. դոկտոր Դ.Սահակյան(ԱԱԳԼ)

Առաջատար կազմակերպություն՝ Երևանի Պետական Համալսարան  
Պաշտպանությունը կայանալու է 2019 թ.մարտի 15 ին ժամը 14-ին, ԱԱԳԼ-ում  
գործող ԲՈՀ-ի 024 «Ֆիզիկայի» մասնագիտական խորհրդում (Երևան - 0036,  
Ալիխանյան եղբայրների փ. 2):

Ատենախոսությանը կարելի է ծանուցանալ ԱԱԳԼ-ի գրադարանում:

Սեղմագիրն ստաբված է 2019 թ. հունվարի 29-ին:

Մասնագիտական խորհրդի գիտական քարտուղար

Ֆիզ. մաթ գիտ. դոկտոր

Հ. Մարուքյան

The subject of the dissertation is approved by the scientific council of the  
A.Alikhanyan National Science Laboratory (AANL)

Scientific advisor: d.ph-math sci.

A Sedrakyan (AANI )

Official opponents:

d.ph-math sci.

R Poghosyan(AANI )

d.ph-math sci.

D Sahakyan(AANI )

Leading organization:

Yerevan State University.

Defense takes place on 15<sup>th</sup> of March at 14 00 on meeting of the  
special council of the Higher Attestation Commission of RA 024 "Physics", acting in

A.Alikhanyan National science Laboratory(Yerevan 0036, st. Alikhanyan Brothers 2)

The dissertation can be learnt at the AANI library

The synopsis is sent out on 29 01 2019

Scientific secretary of the special council

Doctor of physical and mathematical sciences.

H.Marukyan

## Abstract

The quantum Hall effect was a constant source of new ideas, most of which are related to how topology invades quantum physics. Attractive examples include topological isolator, topological order, and topological quantum computations. Basically, all of these phenomena are an impressive theoretical construct that involves traveling through some of the most fascinating and important developments in the field of theoretical and mathematical physics in recent decades.

The first attack on the problem focussed on the microscopic aspects of the electron wavefunctions. Subsequent approaches looked at the system from a more coarse-grained, field-theoretic perspective where a subtle construction known as Chern-Simons theory plays the key role. Yet another perspective comes from the edge of the sample where specific excitations. Graphene now is attracting scientists with its peculiar material characteristics. Electrons in graphene strongly interact and therefore exhibit fractional quantum Hall effect. But it is remarkable that evidence of the collective behavior of electrons in graphene is still lacking.

The integer quantum Hall effect can be described only in terms of individual electrons in a magnetic field while the fractional quantum Hall effect can be understood by studying the collective behaviour of all the electrons. The quantum Hall effect is also studied in context of conform field theory. Recently, it has been proposed that the order parameter of the fractional quantum Hall effect is related to the vertex operator, and the ground state wave the function of a certain fractional filling factor can be expressed in terms of the N-point correlation function of vertex operator.

The application of conformal field theory has thus been extended into a rather specific condensed matter phenomenon. The Laughlin states for N interacting electrons at the plateaus of the fractional Hall effect are examined in the thermodynamic limit of large N.

The quantum Hall systems now is a major paradigm in condensed matter physics, with important applications such as resistance metrology and measurements of fundamental constants. In the recent years, it has been shown that the quantum Hall effect is just one member of a much larger family of topologically specific quantum states, some of which contain the quantum spin Hall effect which is also known as the 2D topological insulator and 3D TI's.

The main task of the thesis is to study some aspects of quantum Hall effect and the relation with Liouville field theory.

### Timeliness and relevance

Graphene is now attracting scientists with its peculiar material characteristics. Electrons in graphene strongly interact and therefore exhibit fractional quantum Hall effect. But remarkably, the evidence for collective behaviour of electrons in graphene still is absent. The integer quantum Hall effect can be described only in terms of individual electrons in a magnetic field while the fractional Hall effect is related to the collective behavior of electrons.

The quantum Hall effect is also studied in the context of conformal field theory. Two-dimensional conformal field theories describe statistical systems at critical points and provide the classical solutions of string theory. Recently, it has been proposed that the order parameter of the fractional quantum Hall effect is related to the vertex operator, and the ground state wave function of a certain fractional filling factor can be expressed in terms of the N-point correlation function of vertex operators. It's important to notice that the Fractional Quantum Hall effect is possible also to describe by the Liouville field theory, whose cosmological constant should play a role of a chemical potential, which plays an important role in quantum Hall effect

The electrical conductivity of graphene, a two-dimensional hexagonal lattice of carbon atoms, possesses many qualities necessary for promising applications in both fundamental physics and nanotechnology. At energies below a few electron volts the electronic properties of graphene are perfectly described by the Dirac model

In a series of papers authors had calculated conductivity with non zero gap, chemical potential, scattering rate and magnetic field. However current response functions were not studied properly in the presence of non quantized external magnetic field and chemical potential

### Aim of the dissertation

- To calculate the current-current correlation function of 3D massive Dirac fermions in the presence of chemical potential
- To study the current-current correlation function in presence of external non quantized magnetic field and chemical potential
- To examine transport properties of topological insulator and analyze polarization operator

- To investigate the boundary Liouville three point function in mini-superspace limit
- To explore topological defects in Liouville field theory with different cosmological constants which is related which can play the role of chemical potential in quantum Hall effect

### Novelty of the work

The basic meaning of the quantum Hall effects (integer and fractional) is that they are examples of physical systems in which quantization effects appear macroscopically.

This results arose from an interesting interplay between disorder, topology and interactions. The important factor is that an electron is in the magnetic field. After solving the Schrödinger equation, so-called Landau levels are obtained, which have quantized energy values separated by a large gap. Actually, this is nothing special, since most quantum-mechanical problems have a discrete spectrum. But in the case of the quantum Hall effect, this discontinuity occurs when we measure the Hall conductivity. For the value of conductivity there are two possibilities that lead to integer quantum Hall and fractional quantum Hall effects.

The significance of quantum Hall effect research is related to modern technologies. There are some classes of materials whose electric conductivity increases with temperature and whose charge carriers can be either positive or negative, depending on the impurity introduced into them these materials are called semiconductors.

### Practical value

- It has been obtained current-current correlation function in 3D massive Dirac theory with chemical potential.
- It has been shown the behaviour of current-current correlation function in third order of Feynman diagrams in the presence of chemical potential and magnetic field .
- It, has been studied the most spectra of topological insulator regarding cold atoms with spin-orbit interacting.
- It has been obtained boundary three point function on mini-superspace in Liouville field theory and also has been computed matrix elements for the

Morse potential quantum mechanics. An exact agreement between the former and latter has been found. We show that both of them are given by the generalized hypergeometric functions.

- We construct topological defects in the Liouville field theory producing jump in the value of cosmological constant. We construct them using the Cardy-Lewellen equation for the two-point function with defect.
- We show that there are continuous and discrete families of such kind of defects.
- For the continuous family of defects we also find the Lagrangian description and check its agreement with the solution of the Cardy-Lewellen equation using the heavy asymptotic quasi-classical limit.

### Main points to defend

In the dissertation we will represent the expression of current-current correlation function in second order of Feynman diagram with chemical potential. For third order Feynman's diagram will be calculated current response function in the presence of chemical potential and magnetic field. For topological insulator with moat spectra we will calculate and analyze the operator of polarization.

Then we will study the mini-superspace limit of the boundary three-point function in the boundary Liouville field theory (BLFT). The boundary three-point function in the BLFT will be computed in and expressed via double Gamma and double Sine functions. Using known asymptotic properties of the double Gamma and Sine functions we will show that in the mini superspace limit the boundary three point function can be expressed via the Meijer functions with the unit argument or equivalently via the generalized hypergeometric functions with the unit argument. Also we will construct topological defects gluing 2D Liouville field theories with different cosmological constants.

### Structure of the dissertation

The dissertation consists of the introduction, five chapters, and finally the list of used literature and pictures.

### Content of the dissertation

In **introduction** we briefly discuss the graphene, some aspects of quantum Hall effect, topological insulators and boundary Liouville field theory with different cosmological constants. We describe the equation of motion in external magnetic field. The fact that magnetic field causes charged particles to move in circles creates the Hall effect in context of Drude model, where the resistivity is defined as the inverse of the conductivity.

We also look at the quantum mechanics of free particles moving against the background of a magnetic field and creating Landau levels. When we have a magnetic field, Zeeman splitting arises between the energies of the spins up and down.

In the presence of a magnetic field, there is a Zeeman splitting between the energies of the spins up and down. In the presence of a magnetic field, the energy levels of the particles become equal from each other, where the gap between each level is proportional to the magnetic field. These energy levels are called Landau levels.

The **first** chapter shows that the reaction of the fermion system to external gauge fields is determined by the current-current correlation function. Transport properties of different physical quantities are determined by zero energy-momentum limit of it. It is well known close to half-filling the physics of graphene is described by (2+1) dimensional Dirac theory.

In this chapter we calculate the current-current correlation function in Dirac theory in the presence of chemical potential  $\mu$  and gap  $m$ .

Since each Dirac point contributes to response function additively, from Dirac action in three-dimensional space-time with chemical potential and gap. We intend to calculate the current-current correlation function for the three-dimensional theory with the kinetic part for the fermions and the interaction term with gauge field in the one-loop approximation.

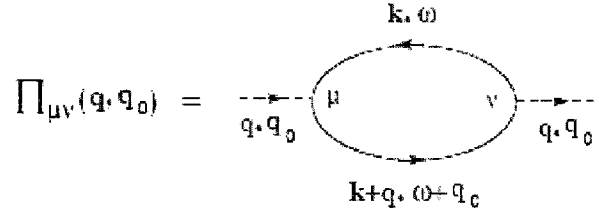


Fig.1. The lower order Feynman diagram for current-current correlation function.

The action which describes the graphene in the Effective Field Theory (EFT) framework via four-component massive Dirac fermions with instantaneous three-dimensional Coulomb interactions is the following (in Euclidean space time)

$$S_g = -\sum_{i=1}^{N_f} \int d^3x dt \bar{\psi}_i (\gamma^0 \partial_0 + v \gamma^k \partial_k + i A_0 \gamma^0 + m) \psi_i + \frac{1}{2g^2} \int d^2x dt (\partial_k A_0)^2.$$

The current-current correlation function is defined by Feynman diagram (see Fig.1).

$$\Pi_{\mu\nu}(\mathbf{r}-\mathbf{r}') = \langle j_\mu(\mathbf{r}) j_\nu(\mathbf{r}') \rangle$$

and in momentum space it reads

$$\Pi_{\mu\nu} = \int_{-x}^x \frac{d^3k}{(2\pi)^3} \text{Tr} [\sigma_\mu G(k) \sigma_\nu G(k+q)] = \int_{-x}^x \frac{d^3k}{(2\pi)^3} \frac{\text{Tr} [\sigma_\mu (k_\rho \sigma_\rho - m) \sigma_\nu (k_\lambda \sigma_\lambda - m)]}{[k^2 + m^2][(k+q)^2 + m^2]}.$$

The four-component fermionic structure is conditioned by the existence of the quasi-particle excitations in two sublattices in the graphene around two Dirac points. Note that for grapheme model's case with four-component fermions, the last term linear by mass must be annihilated due to the contributions of two different two-dimensional fermions with opposite parities. The expression of Trace combined with the formula of Feynman parameterization. The shift of integration energy/momentum gives

$$\Pi_{\mu\nu} = 2 \int_{-x}^x \frac{d^3k}{(2\pi)^3} \int_0^1 dx \frac{2k^\rho k^\lambda - \delta^{\rho\lambda} (k^2 + m^2 + q^2 x(1-x)) + 2x(1-x)(\delta^{\rho\nu} q^2 - q^\rho q^\nu)}{(k^2 + m^2 + q^2 x(1-x))^2}$$

$$-im \epsilon_{\mu\nu\rho} q_\rho \int_{-\infty}^{\infty} \frac{d^3k}{(2\pi)^3} \int_0^1 dx \frac{1}{(k^2 + m^2 + q^2 x(1-x))^2} = \Pi_{\mu\nu}^{(1)} + \Pi_{\mu\nu}^{(2)} + \Pi_{\mu\nu}^{(3)}.$$

Thus, three terms are chosen in the last equation in this way: the first is the part that does not satisfy the condition of charge conservation. The third term is the anomaly part. Second, the main transversal term is

$$\Pi_{\mu\nu}^{(2)} = 2(\delta^{\mu\nu} q^2 - q^\rho q^\nu) \int_{-\infty}^{\infty} \frac{d^3k}{(2\pi)^3} \int_0^1 dx \frac{2x(1-x)}{(k^2 + m^2 + q^2 x(1-x))^2}$$

Then, for  $q^2 \geq 4(\eta^2 - m^2) \geq 0$  when the square root in the expression of is real, the integral gives

$$\Pi_{\mu\nu}^{(2)} = \frac{(\delta^{\mu\nu} q^2 - q^\rho q^\nu)}{\pi} \times \left[ \frac{1}{12\eta} \left( 1 + \left( \frac{\eta^2 + 2m^2}{q^2} - 1 \right) \sqrt{1 - \frac{4(\eta^2 - m^2)}{q^2}} \right) + \frac{1 - \frac{4m^2}{q^2}}{8q} \arctan \frac{q \sqrt{1 - \frac{4(\eta^2 - m^2)}{q^2}}}{2\eta} \right]$$

In the opposite region  $4(\eta^2 - m^2) \geq q^2$  the integral becomes

$$\Pi_{\mu\nu}^{(2)} = \frac{(\delta^{\mu\nu} q^2 - q^\rho q^\nu)}{12\pi\eta}.$$

For the case  $(\eta^2 - m^2) \leq 0$  the expression for  $x_{1,2}$  defines larger than segment  $[0,1]$  region and we have to put  $x_1 = 0, x_2 = 1$ . We have a result

$$\Pi_{\mu\nu}^{(2)} = \frac{(\delta^{\mu\nu} q^2 - q^\rho q^\nu)}{\pi} \frac{1}{8q} \left( \frac{2m}{q} + \left( 1 - \frac{4m^2}{q^2} \right) \arctan \frac{q}{2m} \right).$$

$$\Pi_{\mu\nu}^{(3)} = -\frac{im q_\rho \epsilon_{\mu\nu\rho}}{2\pi q} \arctan \left[ \frac{q}{2|m|} \right] = -\frac{i}{4\pi} \text{sign}[m] q_\rho \epsilon_{\mu\nu\rho} + O\left(\frac{q^2}{m^2}\right)$$

The result for the grapheme with two Dirac fields of opposite chirality is  $\Pi_{\mu\nu_z} = 2\Pi_{\mu\nu}^{(2)}$  because the anomaly term will be cancelled.

In **second** chapter we calculate the response of fermionic system to external gauge fields in presence of non-quantized magnetic field is determined by current-current correlation function. We study 2D dimensional Dirac electron system and calculate current-current correlation function in a presence of magnetic field  $B$ , chemical potential and gap. The magnetic field dependence of the current-current correlation function is defined by third order Feynman diagrams in Fig. 2. For current-current correlation function we get

$$\Pi_{\mu\nu_z}(B) = -\frac{iB}{4\pi|\eta|} \epsilon_{\mu\nu} q_\nu (\Gamma + i\eta) \left( \frac{1}{m^2 + \frac{q^2}{4}} \sqrt{1 - \frac{4(\eta^2 - m^2)}{q^2}} + \frac{1}{\eta^2} \left( 1 - \sqrt{1 - \frac{4(\eta^2 - m^2)}{q^2}} \right) \right)$$

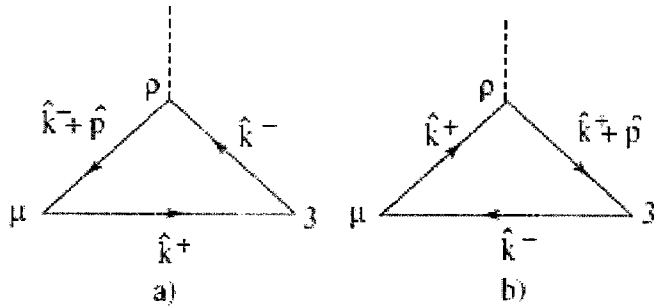


Fig.2. Third order Feynman diagram for current-current correlation function

The **third** chapter is about topological insulators. Topological insulators (TIs) are a class of such materials, and nowadays they are a hot topics for the research.

As TIs are related to a new side of quantum mechanics, the topology of the Hilbert space, they brought a new understanding to the considered workings of nature.

TIs are called "topological" because the wave functions representing their electronic state range a Hilbert space that has a nontrivial topology. Quantum-mechanical wave functions are characterized by linear combinations of orthonormal vectors creating a basis set, and the abstract space spanned by this orthonormal basis is called Hilbert

space. In crystalline solids, the wave function can be viewed as a mapping from the  $k$ -space to a manifold in the Hilbert space (or in its projection), and hence the topology becomes relevant to electronic states in the solids.

Depending on the way the Hilbert-space topology becomes nontrivial, there can be various different kinds of TIs.

An important consequence of the nontrivial topology related to the wave functions of the insulator is that the gapless interface state necessarily manifests itself when the insulator physically ends and collides with a conventional insulator (including vacuum).

This happens because the nontrivial topology is a discrete characteristic of gapped energy states, and as long as the energy gap remains open, the topology cannot change; hence, in order the topology to change across the interface into a trivial one, the gap must be closed at the interface. Therefore, three-dimensional 3D TIs are always connected with the gapless surface states, and so are two-dimensional (2D) TIs with gapless edge states. This necessity condition for the occurrence of gapless interface states is called bulk-boundary correspondence in topological phases.

As for the two-dimensional Dirac fermions, they can also be observed on the surface of three-dimensional topological insulators.

Although they are doubled, it is easier to control them, because they occur on opposite surfaces that are spatially separated.

Some experimental and theoretical studies on the QHE of the surface states of topological insulators have been examined. Particularly, in a magnetic topological insulator with broken inversion symmetry nondegenerate surface states have been realized and the QHE for a single Dirac fermion has been observed.

In third chapter the polarization operator of non-relativistic fermions with spin-orbit (SO) Rashba interaction is presented. The spectrum of this fermions is most type having minimum on a circle. Contrary to the Dirac or non-relativistic fermions Fermi sea, here there is a geometry of Corbino disk which reflects on a transport properties of excitations. In modern physics there are materials, such as topological insulators (TI), where the spectrum of non-relativistic particles is combined with relativistic Dirac component.

The analysis of the transport properties of this type of system is an important task that needs to be solved.

The polarization operator is a mathematical construction that determines both the longitudinal and Hall conductivities on the one hand and the effective action of the gauge field defined by quantum fluctuations of fermions, on the other. In this

chapter, the calculation of the polarization operator of fermions with a spectrum of the type is presented. The most common form of the main Hamiltonian of such systems is

$$H(k) = \epsilon_k + \sum_{i=x,y,z} d_i(k) \sigma_i$$

Characteristics picture of two branches of this spectrum is presented in Fig.3, while only lower branch is forming the ground state.

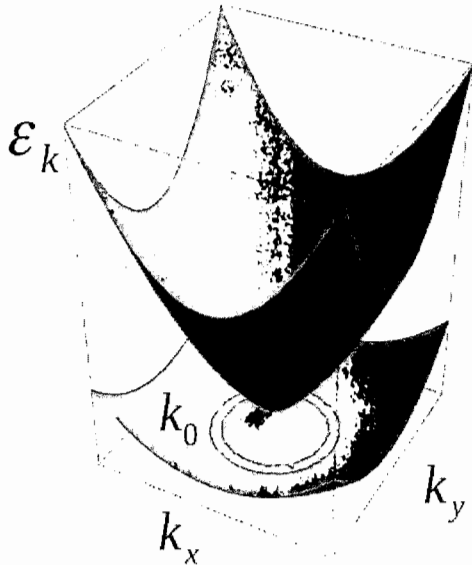


Fig.3. Two branches of spectra of moat type

In Fig.4a the characteristic form of the Fermi sea is presented

In result for longitudinal conductivity as a coefficient of linear response to external electric field is obtained

$$\sigma_{xx}(\omega) = \frac{T_{xx}(k_{1F}, 0) - T_{xx}(k_{2F}, 0)}{8\pi \left( \frac{1}{m} - \frac{v_F^2}{\epsilon_k} \right)} \frac{i}{\omega + 2i\eta}$$

We have presented here the calculation of the polarization operator in the fermionic system, which have moat type spectrum. The answer has normal part, leading to Drude conductivity and anomaly part, defined by Rashba SO-term in the Hamiltonian. The result presents correct, expected limits at  $v_F = 0$ , when Rashba term is zero, at  $m = \infty$ , when non-relativistic part of the Hamiltonian is zero and we have only Rashba term. At the minimal chemical potential the conductivity became zero.

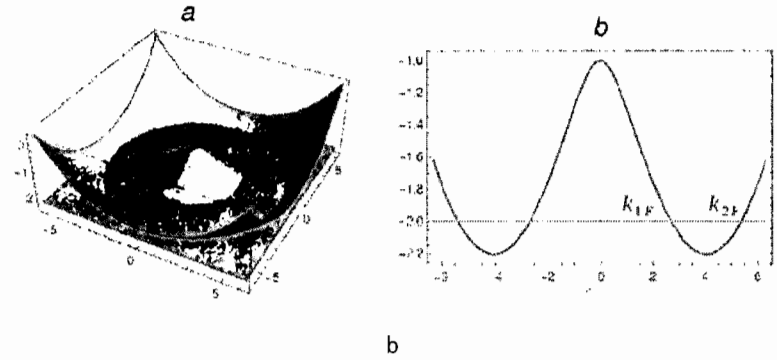


Fig.4. (a) Lower branch of spectrum with filled Fermi sea. (b) The Fermi momentum of inner and outer circles

In **fourth** chapter it is presented the mini-superspace limit of boundary three-point function in Liouville field theory. In this chapter the matrix elements of the boundary Liouville field theory are studied in the mini-superspace limit. In the minisuperspace limit only the zero mode dynamics survives and the theory is

reduced to the corresponding quantum mechanical problem. The mini-superspace limit of the Liouville field theory was considered in some works. In this chapter we study the mini-superspace limit of the boundary three-point function in the BLFT. The boundary three-point function in the BLFT was computed and expressed via double Gamma and double Sine functions. Using known asymptotic properties of the double Gamma and Sine functions, we have shown that in the mini-superspace limit the boundary three-point function can be expressed via the Meijer functions with the unit argument or equivalently via the generalized hypergeometric functions with the unit argument. Also the matrix elements are computed for the Morse potential and have shown that they can be expressed via the generalized hypergeometric functions with the unit argument as well. Using the identities, relating different generalized hypergeometric functions with the unit argument, and matching quantum and classical parameters, we established exact agreement between the mini-superspace limit of the boundary three-point function and the matrix elements for the Morse potential. It is important to note that in the BLFT relation of the boundary cosmological parameter to the corresponding quantum parameter appearing in the boundary one-point function is two-fold due to a sign ambiguity in the choice of the square-root branch. It has been found that to match the minisuperspace limit of the boundary three-point function with the corresponding quantum mechanical matrix element

we should use the branch with the negative sign. In the thesis we also show that the passing from one branch to the another one brings to additional factor in the normalization of the wave functions corresponding to the boundary condition changing operators. We would like also to mention that various consequences of the branching of the BLFT parameters earlier were considered in some publications.

In the mini-superspace limit the boundary Liouville field theory is described by the Hamiltonian with the Morse potential. The corresponding eigenfunctions satisfy the Schrodinger equation. It has been discussed in this chapter quasi-classical properties of the boundary three-point functions. We found perfect agreement with the corresponding quantum mechanical calculations. The matching of the calculations required to consider the negative branch in the branched correspondence of the classical and quantum parameters. It has been shown that passing from one branch to another leads to the change in the normalization of the wave functions. It has been found the flip of the boundary conditions induced by the exponential operators in the minisuperspace limit.

In **fifth** chapter it is constructed the topological defect in the Liouville field theory producing jump in the value of cosmological constant. We construct it using the

Cardy-Lewellen equation for the two-point function with defect. It has been shown that there are continuous and discrete families of such kind of defects. For the continuous family of defects we found the Lagrangian description and check its agreement with the solution of the Cardy-Lewellen equation using the heavy asymptotic semiclassical limit. The connection between the fractional Hall effect and two-dimensional conformal field theories is well known and has many different aspects. Much effort was put into establishing connections between various conformal blocks and Laughlin's wave functions.

Cappelli with collaborators studied the Hall effect using incompressible fluid approach with symmetry. In were considered conformal blocks and their relation to Hall effect on arbitrary Riemann surface. Another very interesting aspect is the relation between edge physics of Hall effect and 2D CFT with boundaries and topological defects. In the recent works emerged important role of 2D Liouville CFT in study of the fractional Hall effect. In these works the Liouville theory cosmological constant plays the role of the chemical potential.

Therefore one can hope that boundary Liouville field theory and Liouville theory with defects can have applications to the edge physics of Hall effect. In the fourth chapter it is studied three-point boundary correlation function in the Liouville field theory. The motivation for the second work was the fact that in the FQHE one has jump of the chemical potential. In the fifth chapter it is constructed topological defect gluing Liouville field theories with different cosmological constant. We have an impression that our construction can be useful to understand the physics of the mentioned jump. It has been checked that the system of the defect equations of motion guarantees that both components of the energy-momentum tensor are continuous across the defects and therefore describes topological defects.

In **conclusion**, the main results of the dissertation are listed.

The complete list of **used literature** is presented in the thesis.

**Dissertation's presentation**

The main results of the thesis were discussed at the YerPhI Joint Theoretical Physics Laboratory and Yerevan State University(YSU).

#### Publication's list

1. **E.Apresyan**, A.Sedrakyan, Sh.Khachatryan, Current-current correlation function in 3D massive Dirac theory with chemical potential, Mod. Phys. Lett. A30 (7): 1550035 (2015).
2. **E.Apresyan**, Current-current correlation function in the presence of chemical potential and magnetic field, Journal of Contemporary Physics (Armenian Academy of sciences), Volume 5, Issue 2 , pp 105-109 (2017).
3. **E.Apresyan**, G.Sarkissian, On mini- superspace limit of boundary three point function in Liouville field theory JHEP 12 (2017) 058.
4. **E.Apresyan**, G.Sarkissian, Topological defects in the Liouville field theories with different cosmological constants JHEP 05 (2018) 131.

## Հուլի քվանտային երևույթի որոշ ասպեկտները Ամփոփում

Դիսերտացիայի մեջ ուսումնասիրվում են Հուլի քվանտային երևույթի որոշ ասպեկտները: Դիտարկվել է 2+1 չափանի Դիրակի անզանգված ֆերմիոններից կազմված համակարգ, որի օրինակ է գրաֆենը: Հաշվվել է հաղորդականությունը գրաֆենում կամայական քիմիական պոտենցիալի առակայության դեպքում Ֆեյնմանի դիագրամի մեկ օղակային մոտավորությամբ: Նման համակարգերը հետաքրքիր են նրանով, որ այդ մոդելներում հնարավոր է դիտել Հուլի քվանտային երևույթ գոյական հաղորդականության և գոյական մագնիսական դաշտի բացակայության դեպքում: Աշխատանքի մեջ ստացվել է արտահայտություն բևեռացման օպերատորի համար, որի հաշվարկը թույլ է տալիս որոշել նույն հաղորդականությունը:

Դիսերտացիայի մեջ հաշվվել է նաև 2+1 չափանի համակարգի համար հոսանք-հոսանք կորելիացիոն ֆունկցիան կամայական քիմիական պոտենցիալի և ոչ-քվանտացված արտաքին մագնիսական դաշտի առկայության դեպքում: Հաշվարկներն իրականացվել են Ֆեյնմանի դիագրամների երրորդ կարգի մոտավորությամբ (3+1 չափանի համակարգերում նման դիագրամներն անհիիլացվում են Ֆարրիի թեորեմի համաձայն):

Ժամանակակից ֆիզիկայում կան որոշ նյութեր, ինչպիսիք են օրինակ տոպոլոգիական մեկուսիչներն եզրային վիճակներով, որոնք մեծ հետաքրքրություն են ներկայացնում: Տոպոլոգիական մեկուսիչների համար հաշվվել է բևեռացման օպերատորը ոչ-ռեյալիստիկ ֆերմիոնների Ռաբբա սպին-ուղեծրային փոխազդեցության առկայության դեպքում: Էներգիական սպեկտրն այդպիսի ֆերմիոնների փոսաձև է և ունի մինիմում շրջանի վրա: Ի հակադրություն Դիրակի կամ ոչ-ռեյալիստիկ ֆերմիոնների այստեղ Ֆերմի «ծովն» ունի Կորբինոյի սկավառակի երկրաչափություն, որն արտահայտում է խտությունների տեղափոխական հատկությունները:

Դիտարկվել են տոպոլոգիական դեֆեկտներ Լիովիլի դաշտի տեսության շրջանակներում կոսմոլոգիական հաստատունի թռիչքով: Նման մոդել կառուցվել է օգտագործելով Կարգի-Լևիլենի հավասարումը դեֆեկտով երկու կետանի ֆունկցիայի համար: Ցույց է տրվել, որ կան դիսկրետ և անընդհատ ընտանիքներ նման դեֆեկտների: Դեֆեկտների անընդհատ ընտանիքի համար գտնվել է Լազրանժիան նկարագրությունը և ստուգվել է համաձայնությունը Կարգի-Լևիլենի հավասարման հետ՝ ծանր ասիմպտոտիկ կիսադասական սահմանում: Կապը Հուլի քվանտային երևույթի և երկու չափանի կոնֆորմ դաշտի տեսության հետ հայտնի և ունի տարբեր ասպեկտներ:

Ուսումնասիրվել են եզրային Լիովիլի տեսության մատրիցական էլեմենտները

մինի-գերտարածական սահմանում, այդ սահմանում մնում են միայն գոյական մոդերը, որի արդյունքում գալիս ենք քվանտամեխանիկական խնդրի: Մինի-գերտարածական սահմանում հաշվվել է եզրային Լիովիլի դաշտի տեսության երեք կետանի ֆունկցիան, որն արտահայտվել է զույգ Գամմա և սինուսոիդային ֆունկցիաների միջոցով: Օգտագործելով ասիմպտոտիկ հատկությունները նշված ֆունկցիաների՝ ցույց է տրվել, որ մինի-գերտարածական սահմանում եզրային երեք կետանի ֆունկցիան կարող է արտահայտվել Մեյերի միավոր արգումենտով կամ էվլիվալենտ ընդհանրացված միավոր արգումենտով հիպերերկրաչափական ֆունկցիաների միջոցով: Հաշվվել են նաև Մորգե պոտենցիալի մատրիցական էլեմենտները և ցույց է տրվել, որ նրանք նույնպես կարող են արտահայտվել համապատասխան հիպերերկրաչափական ֆունկցիաներով:

## Некоторые аспекты явления квантового Холла Резюме

В диссертации изучаются некоторые аспекты эффекта квантового Холла. Была рассмотрена (2+1)-мерная система состоящая из дираковских безмассовых фермионов, примером которого является графен. Проводимость была рассчитана для графена при произвольном химическом потенциале в приближении однопетлевой диаграммы Фейнмана. Такие системы интересны тем, что в этих моделях можно наблюдать явление квантового эффекта Холла при отсутствии нулевой проводимости и нулевого магнитного поля. В работе была получена формула для поляризационного оператора, расчет которой позволяет определить проводимость.

В диссертации рассчитана также ток-ток корреляционная функция для 2+1-мерной системы в случае произвольного химического потенциала и неквантованного внешнего магнитного поля. Расчеты проводились в приближении третьего порядка диаграмм Фейнмана (3 + 1 мерных системах такие диаграммы не учитываются согласно теореме Фарри). В современной физике существуют некоторые вещества, например топологические изоляторы с конечными состояниями, которые представляют большой интерес. Для топологических изоляторов рассчитан оператор поляризации для спин-орбитального взаимодействия Рашбы с нерелятивистскими фермионами. Энергетический спектр таких фермионов представляет собой яму и имеет минимальную окружность.

В отличие от дираковских или нерелятивистских фермионов, море Ферми имеет геометрию диска Корбино, которая отражает транспортные свойства возбуждений. Рассмотрены топологические дефекты в теории Лиувилля с различными космологическими константами. Аналогичная модель была построена с использованием уравнения Карди-Левилена с дефектом для двухточечной функции. Показано, что существуют дискретные и постоянные семейства с такими дефектами. Для непрерывного семейства дефектов было найдено описание лагранжиана и показано согласие с уравнением Карди-Левилена в тяжелом квазиклассическом пределе. Взаимосвязь между квантовым эффектом Холла и двумерной конформной теорией поля имеет различные аспекты. Математические элементы граничной теории Лиувилля изучались в рамках мини-суперпространстве. В этом пределе остаются только нулевые моды, и в результате мы приходим к квантово-механической задаче. Матричные элементы граничной теории Лиувилля изучались в мини-суперпространств, где остаются только нулевые моды, и в результате мы приходим к квантово-механической задаче. В мини-суперпространстве была рассчитана предельная

трехточечная функция теории поля Лиувилля, которая была выражена через четную Гамма-функции и синусоидальные функции. Используя асимптотические свойства этих функций, было показано, что конечная трехточечная функция в пределе мини-суперпространства может быть выражена через функцию Мейера единичным аргументом или через эквивалентную обобщенную гипергеометрическую функцию единичным аргументом. Матричные элементы потенциала Морзе также были вычислены и показаны, что они также могут быть выражены в соответствующих гиперметрических функциях.

В.И.И.